A PROPOSITION IN REFERENCE TO CENTRE OF GRAVITY, AND ITS DEMONSTRATION.

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PROPOSITION. The point $P'(x',y',z')$ is the centre of gravity of the mass m if the sum (s) of the squares of the distances from P' to every point of m, is a minimum.

PROOF. Let $P(x,y,z)$ be any point of m, then the square of the distance PP' is $PP'^2=(x-x')^2+(y-y')^2+(z-z')^2$,

$$\text{and } s=\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2}[(x-x')^2+(y-y')^2+(z-z')^2]dxdydz.$$

Representing $\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2}$ by $\int$ and $dxdydz$ by dm,

$$\text{we have } s=\int[(x-x')^2+(y-y')^2+(z-z')^2]dm.$$

Since s is a minimum with respect to the independent variables, x',y',z', we have $\frac{ds}{dx'}=0$, $\frac{ds}{dy'}=0$, and $\frac{ds}{dz'}=0$; that is,

$$(1).\quad \int(x-x')dm=0, \quad \therefore \quad x'=\frac{\int xdm}{\int dm};$$

$$(2).\quad \int(y-y')dm=0, \quad \therefore \quad y'=\frac{\int xdm}{\int dm};$$

$$(3).\quad \int(z-z')dm=0, \quad \therefore \quad z'=\frac{\int zdm}{\int dm}$$

Q. E. D.